# Problem 1

## Part A

Determine the number of refrigerators to be shipped plants to warehouses and then warehouses to retailers to minimize the cost.

1. Formulate the problem as a linear program with an objective function and all constraints.
2. Determine the optimal solution for the linear program using any software you want. Include a copy of the code/file in the report.
3. What are the optimal shipping routes and minimum cost.

### Part A-i

**Define Variables:**

Let be the Plants {P1, P2, P3, P4}

Let be the Warehouses {W1, W2, W3}

Let be the Retailers {R1, R2, R3, R4, R5, R6, R7}

Let be the units to ship from to

Let be the units to ship from to

Let be the supply available at each plant

Let be the demand at each retailer .

**Objective:** Minimize the costof shipping the products from the plants to the warehouses and warehouses to the retailers.

MINIMIZE

**Constaints:**

s.t.:

// Each Plant cannot ship more than it has

//p1 supply

//p2 supply

//p3 supply

//p4 supply

// Each retailer receives exactly what its demand is

// Retailer 1 demand

// Retailer 2 demand

// Retailer 3 demand

// Retailer 4 demand

// Retailer 5 demand

// Retailer 6 demand

// Retailer 7 demand

// Units shipped from each warehouse must be <= amount shipped to the warehouse

**Non-negativity constraints:**

and

### Part A-ii

The optimal solution was generated using the above formulas translated into a LINDO program.

Code is in file program3.ltx as well as below. The output from LINDO follows the code.

MINIMIZE 10X11 + 15X12 + 11X21 + 8X22 + 13X31 + 8X32 + 9X33 + 14X42 + 8X43 + 5Y11 + 6Y12 + 7Y13 + 10Y14 + 12Y23 + 8Y24 + 10Y25 + 14Y26 + 14Y34 + 12Y35 + 12Y36 + 6Y37

ST

X11 + X12 <= 150

X21 + X22 <= 450

X31 + X32 + X33 <= 250

X42 + X43 <= 150

Y11 = 100

Y12 = 150

Y13 + Y23 = 100

Y14 + Y24 + Y34 = 200

Y25 + Y35 = 200

Y26 + Y36 = 150

Y37 = 100

Y11 + Y12 + Y13 + Y14-X11-X21-X31 <= 0

Y23 + Y24 + Y25 + Y26-X12-X22-X32-X42 <= 0

Y34 + Y35 + Y36 + Y37-X33-X43 <= 0

Results:

LP OPTIMUM FOUND AT STEP 10

OBJECTIVE FUNCTION VALUE

1) 17100.00

VARIABLE VALUE REDUCED COST

X11 150.000000 0.000000

X12 0.000000 8.000000

X21 200.000000 0.000000

X22 250.000000 0.000000

X31 0.000000 2.000000

X32 150.000000 0.000000

X33 100.000000 0.000000

X42 0.000000 7.000000

X43 150.000000 0.000000

Y11 100.000000 0.000000

Y12 150.000000 0.000000

Y13 100.000000 0.000000

Y14 0.000000 5.000000

Y23 0.000000 2.000000

Y24 200.000000 0.000000

Y25 200.000000 0.000000

Y26 0.000000 1.000000

Y34 0.000000 7.000000

Y35 0.000000 3.000000

Y36 150.000000 0.000000

Y37 100.000000 0.000000

ROW SLACK OR SURPLUS DUAL PRICES

2) 0.000000 1.000000

3) 0.000000 0.000000

4) 0.000000 0.000000

5) 0.000000 1.000000

6) 0.000000 -16.000000

7) 0.000000 -17.000000

8) 0.000000 -18.000000

9) 0.000000 -16.000000

10) 0.000000 -18.000000

11) 0.000000 -21.000000

12) 0.000000 -15.000000

13) 0.000000 11.000000

14) 0.000000 8.000000

15) 0.000000 9.000000

NO. ITERATIONS= 10

### Part A-iii:

Optimal shipping routes and minimum cost can be determined by examining the results from the program:

**The minimum cost is 17,100.**

**The shipping routes are as follows:**

* P1 ships 150 units to W1
* P2 ships 200 units to W1, 250 units to W2
* P3 ships 150 units to W2 and 100 units to W3
* P4 ships 150 units to W3
* (W1 has 150+200=350 units. W2 has 250+150=400 units. W3 has 100+150=250 units)
* W1 ships 100 to R1, 150 to R2, 100 to R3. (350 units out - OK)
* W2 ships 200 to R4 and 200 R5 (400 total units out - OK)
* W3 ships 150 to R6 and 100 to R7 (250 units out OK).

## Part B

We can solve this question with some simple logic. By eliminating warehouse two, we see the following simple facts:

* Warehouse 3 can receive product only from P3 and P4. That’s a maximum of 400 units of supply coming in to warehouse 3 (250+150).
* The demand from retailers 5, 6, and 7 to receive from Warehouse3 is a total of 450. But there is no way for warehouse 3 to get 450 units. Those three retailers have no other warehouse if warehouse 1 is shut down.

There is no *feasible solution* to this problem, therefore it is “unfeasible”.

We can verify this by running a modified version of the Linear programing that removes warehouse 2:

MINIMIZE

**Constaints:**

s.t.:

// Each Plant cannot ship more than it has

//p1 supply

//p2 supply

//p3 supply

//p4 supply

// Each retailer receives exactly what its demand is

// Retailer 1 demand

// Retailer 2 demand

// Retailer 3 demand

// Retailer 4 demand

// Retailer 5 demand

// Retailer 6 demand

// Retailer 7 demand

// Units shipped from each warehouse must be <= amount shipped to the warehouse

**Non-negativity constraints:**

and

The LINDO code:

MINIMIZE 10X11 + 11X21 + 13X31 + 9X33 + 8X43 + 5Y11 + 6Y12 + 7Y13 + 10Y14 + 14Y34 + 12Y35 + 12Y36 + 6Y37

ST

X11 <= 150

X21 <= 450

X31 + X33 <= 250

X43 <= 150

Y11 = 100

Y12 = 150

Y13 = 100

Y14 + Y34 = 200

Y35 = 200

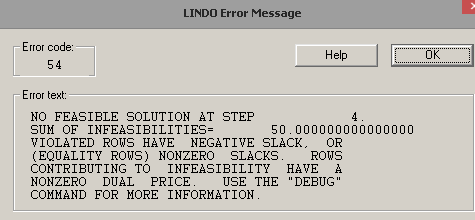
Y36 = 150

Y37 = 100

Y11 + Y12 + Y13 + Y14-X11-X21-X31 <= 0

Y34 + Y35 + Y36 + Y37-X33-X43 <= 0

**Results:**



## Part C

Yes, it is possible if we limit warehouse 2 to having only 100 refrigerators per week. The optimal solution is cost 18300.00.

MINIMIZE

**Constaints:**

s.t.:

// New added constraint: Amount shipped to Warehouse 2 must be no greater than 100 units

// Each Plant cannot ship more than it has

//p1 supply

//p2 supply

//p3 supply

//p4 supply

// Each retailer receives exactly what its demand is

// Retailer 1 demand

// Retailer 2 demand

// Retailer 3 demand

// Retailer 4 demand

// Retailer 5 demand

// Retailer 6 demand

// Retailer 7 demand

// Units shipped from each warehouse must be <= amount shipped to the warehouse

**Non-negativity constraints:**

and

**LINDO CODE (adds one constraint to Part A code) is in problem3c.ltx**

MINIMIZE 10X11 + 15X12 + 11X21 + 8X22 + 13X31 + 8X32 + 9X33 + 14X42 + 8X43 + 5Y11 + 6Y12 + 7Y13 + 10Y14 + 12Y23 + 8Y24 + 10Y25 + 14Y26 + 14Y34 + 12Y35 + 12Y36 + 6Y37

ST

X12 + X22 + X32 + X42 <= 100

X11 + X12 <= 150

X21 + X22 <= 450

X31 + X32 + X33 <= 250

X42 + X43 <= 150

Y11=100

Y12=150

Y13 + Y23=100

Y14 + Y24 + Y34=200

Y25 + Y35=200

Y26 + Y36=150

Y37= 100

Y11 + Y12 + Y13 + Y14-X11-X21-X31 <= 0

Y23 + Y24 + Y25 + Y26-X12-X22-X32-X42 <= 0

Y34 + Y35 + Y36 + Y37-X33-X43 <= 0

**Results**

LP OPTIMUM FOUND AT STEP 11

OBJECTIVE FUNCTION VALUE

1) 18300.00

VARIABLE VALUE REDUCED COST

X11 150.000000 0.000000

X12 0.000000 8.000000

X21 350.000000 0.000000

X22 100.000000 0.000000

X31 0.000000 4.000000

X32 0.000000 2.000000

X33 250.000000 0.000000

X42 0.000000 9.000000

X43 150.000000 0.000000

Y11 100.000000 0.000000

Y12 150.000000 0.000000

Y13 100.000000 0.000000

Y14 150.000000 0.000000

Y23 0.000000 7.000000

Y24 50.000000 0.000000

Y25 50.000000 0.000000

Y26 0.000000 4.000000

Y34 0.000000 4.000000

Y35 150.000000 0.000000

Y36 150.000000 0.000000

Y37 100.000000 0.000000

## Part D

Write out a generalized linear programming model. Give the objective function and constraints as mathematical formula.

For a given Plant-Warehouse-Retailer distribution model with plants, warehouses, and retailers:

Let be the subscript index for plants

Let be the subscript index for Warehouses

Let be the subscript index for Retailers

Let be the units to ship from to

Let be the units to ship from to

Let be the supply available at

Let be the demand at each.

Let be the cost of shipping from to .

Let be the cost of shipping from to

Objective:

MINIMIZE

Constraints:

s.t

// Each Plant cannot ship more than it has

for all i=1 to n and j connected to ?? // TODO FINISH THIS PRLBEM

// Each retailer receives exactly what its demand is

// Retailer 1 demand

// Retailer 2 demand

// Retailer 3 demand

// Retailer 4 demand

// Retailer 5 demand

// Retailer 6 demand

// Retailer 7 demand

// Units shipped from each warehouse must be <= amount shipped to the warehouse

**Non-negativity constraints:**

and